A Simple Model for Financial Aid in Currency Crisis

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Abstract

In most currency crises, it is not uncommon to see countries with problem getting financial supports from other countries. Why countries are interested in rescuing other countries from the viewpoint of economics?

This paper tries to explain the financial aid with a simple two-country model. Consider two central banks compete for export sales to an external market, each facing a symmetric social loss function comprising output stability and price-level stability. Since export affects output, the social loss of a central bank is inevitably affected by another central bank's exchange rate policy. Such relationship, in turn becomes a motivation for a central bank to provide financial aid to intervene another central bank's exchange rate policy decision.

Realignment cost, the fixed cost incurred in abandoning fixed exchange rate policy, is a key determinant of the financial aid. Even though high realignment cost seems to provide greater motivation for a central bank to defend another country's fixed exchange rate in order to maintain its own, this paper reveals that greatest amount of financial aid is provided by a central bank with intermediate instead of high realignment cost.

Some special features of the model are also discussed in the paper. They include the effect of the threat of competitive devaluations in sustaining the exchange rate policy cooperation, the non-monotonic relationship between output sensitivity to the depreciation rate/adjusted inflation-output stability preference and policy response and how speculators can force a central bank to abandon its fixed exchange rate policy indirectly through its export competitor.

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摘要

Abstract (Chinese Version)

在過去的貨幣危機之中，出現危機的一國獲得其他國家的經濟援助(financial aid)並非罕見。從經濟角度出發，各國因何會於貨幣危機中挽救他國?

本文嘗試籍一個簡單的「雙邊模型」(two-country model)闡釋此等經濟援助。假設兩國爭奪同一出口市場，而一國之產出穩定性與價格穩定性構成社會損失(social loss)。基於出口影響產出，一國之社會損失因而受到他國之匯率政策所影響。如此關係給予一國透過提供經濟援助，從而協助他國維持固定匯率政策的動機。

棄守固定匯率政策涉及固定成本，亦即調整成本(realignment cost)，而此成本對於經濟援助之考量至爲緊要。雖然愈高調整成本看似促使一國更有動機援助他國，從而維護自身的固定匯率，惟本文發現，一國在面對中等而非高昂調整成本時，方會提供最大額度的經濟援助。

本文同時討論此模型的其他特性，包括來自「爭相貶值」(competitive devaluations)的威脅對於維繫兩國共同協調匯率政策的作用、出口對於貨幣貶值的敏感度/央行對於產出-通脹偏好與一國的匯率政策反應的非單向性及投機者怎樣透過影響一國出口競爭對手的匯率政策，從而迫使其放棄固定匯率政策。

《經濟文獻雜誌》分類系統編號: F3;F4

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List of Important Notations

L_i^{ij}	The social loss of country i under the interaction of domestic exchange rate policy (i) and foreign exchange rate policy (j).
y_i	Log of country i's output.
$\bar{y}$	Lof of the central bank's target level of output.
ε	The central bank's relative preference on price-level stability and output stability.
p_i	Inflation rate of country i.
e_i	Depreciation rate of country i's currency
α	The sensitivity of country i's output to its relative depreciation rate.
β	The sensitivity of country i's inflation rate to the depreciation rate of its currency.
γ	The central bank's inflation-output stability preference adjusted by the degree of pass-through (i.e. $\gamma = \beta^2\varepsilon$).
C_i	The realignment cost involved in abandoning the fixed exchange rate by country i.
$\bar{S}$	The maximum size of speculative attack.
S_i^f	The size of speculative attack on country i's fixed exchange rate under a foreign fixed exchange rate policy.
S_i^l	The size of speculative attack on country i's fixed exchange rate under a foreign flexible exchange rate policy.
λ^f	The optimization benefit under a foreign fixed exchange rate policy.
λ^l	The optimization benefit under a foreign flexible exchange rate policy.
F	Log of the financial aid

List of Table and Figures

1. Introduction

Global economy has been enjoying satisfactory growth in the past few years. Countries with fixed exchange rate are building up reserves and currency crisis seems to be a far away scenario. From the past experience, however, it is clear that currency crises keep repeating in the history, from the downfall of Bretton Woods system in 1971 to the recent Asian financial crisis in 1997.

Currency crises can have great, if not huge, destructive impact on the domestic as well as global economy. Currency crises usually come with debt crises, bank runs, substantial capital outflow and recession. Countries may take several years to recover from it and their trends of economic growth can be permanently below their pre-crisis level[1].

Currency crises have been studied both theoretically and empirically. Empirical data are tested to identify symptoms of a currency crisis and develop early warning system. On the theoretical side, currency crises models are basically classified into two types, one based on economic fundamentals while another is self-fulfilling. The first-generation model introduced by Krugman (1979) put the causes of crises on weak economic fundamentals and inconsistent macroeconomic policies. Later models pioneered by Obstfeld (1994) argued that exchange rate policy is a tool to minimize social loss and attributed crises to the change of investors' expectation.

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[1] See Hong and Tornell, 2003, "Recovery from a currency crisis: some stylized facts", *Journal of Development Economics* 76, 71-96

Traditional theories mostly focus on stand-alone currency crises. In the real world, however, currency crises are usually contagious. In the Asian crisis, currencies of other countries in the region such as Malaysia, Philippines collapsed shortly after the fall of Thai baht. EMS crisis, Latin American currency crisis and many other currency crises in history showed this similar feature.

Modeling a country's exchange rate policy decision as social loss minimization sheds light on the explanation of contagious currency crisis. Kasa and Huh (2001) interrelated costs of maintaining the fixed exchange rate of different central banks through export competition. For a group of export competing countries, devaluation of a currency weakens the competitiveness of other countries' export and thus increases the costs of maintaining fixed exchanges rates for other central banks.

Kasa and Huh (2001) have an interesting implication. As devaluation of foreign currency increases the costs for domestic central bank, it is clear that domestic central bank should have an intention to prevent it from happening. One of the ways is to provide financial aid to assist foreign central bank to defend its currency. In fact, financial aid is not uncommon in currency crises. During Asian Financial Crisis, Hong Kong joined several countries in Asia and International Monetary Fund (IMF) to provide a US$ 16 billion financial aid to Thailand, the first country under speculative attack in the crisis.

Most existing studies of currency crisis focus on the cause of currency crisis and equilibrium of a country's exchange rate regime under speculative attack[2]. This paper, on the other hand, tries to study and explain financial aid in currency crises. This paper basically follows the framework of Kasa and Huh (2001) to develop a game for the interaction of two central bank's exchange rate policy. Based on the game, this paper further explains and illustrates how financial aid can be used as a tool to minimize social loss of central banks.

A central bank's decision to abandon its fixed exchange rate policy has negative impact on another central bank. To the extreme, the other central bank may be forced to abandon its fixed exchange rate policy as well, thus lead to competitive devaluations between two central banks. Intuitively, a central bank involves high fixed cost (i.e. realignment cost) in abandoning its fixed exchange rate policy should be willing to provide more financial aid to another central bank. However, this paper reveals that a central bank with high realignment cost may actually offer less financial aid. It is because realignment cost of a central bank has two effects on the financial aid decision. Firstly, it increases with the social loss of a central bank in abandoning the fixed exchange rate policy which motivates a central bank to provide more financial aid. Secondly, higher realignment cost reduces the likelihood of an abandonment decision, restraining the amount of financial aid that a central bank is willing to provide. At certain threshold, further increase in realignment cost will reduce a central bank's motivation to provide financial aid.

Rest of the paper is organized as follows. Existing currency crises models are reviewed in the next section. The basic model setup and payoff are introduced in

[2] See, for example, Morris ad Shin (1998) and Angeletos et al (2006).

the following section. Based on the model, two-stage game is developed to discuss the interaction of two central bank's exchange rate policies and important features of the model in such game. Finally, the game is further extended to explain the financial aid decision.

2. Literature Reviews

2.1. Economic Fundamentals Models

The first-generation model, founded by Krugman (1979) and further illustrated by Flood and Garber (1984) suggests that currency crises are caused by inconsistent macroeconomic policies, that is a fixed exchange rate regime and expansionary monetary policy.

The main idea of first-generation model can be presented in a simplified scenario. In a small open economy, purchasing power parity and uncovered interest parity hold and investors have perfect foresight. Government reserve is composed of two assets, domestic and foreign currency. Government is running fiscal deficit, which is financed by increasing domestic currency supply. Under a fixed exchange rate regime without any capital control, government must stand ready to provide foreign currency at a fixed rate. Therefore, any increase in domestic currency supply in excess of its demand is converted into foreign currency by domestic residents. It is clear that a fiscal deficit financed by printing domestic currency will results in losing of foreign currency reserves under a fixed exchange rate regime. The foreign currency reserve will be exhausted by continuous deficit and the government is then forced to float its currency.

The transition of exchange rate from fixed to flexible is not smooth. Instead, there will be a discrete jump in exchange rate at the exhaustion of foreign currency reserve. Such discrete jump means unlimited profit for investors, which should not happen under prefect foresight. Therefore, the fixed exchange rate does not collapse when the foreign currency reserve is exhausted, but when shadow exchange rate (exchange rate if the currency floats immediately) just exceeds the fixed rate. At this point, investors will acquire the rest of foreign currency reserve from the government, force the currency to float and make profit from the difference between two rates. Investors, or often named as speculators by countries under attack, play an important role in currency crises.

It is not difficult to realize that the first-generation model is actually an application of Mundell's impossible trinity[3]. Among a fixed exchange rate, free capital movement and an independent monetary policy, government can only have two, but not all. Under a fixed exchange rate without capital control, it is impossible to use monetary policy to finance fiscal deficit. Otherwise, such monetary policy will finally lead to a collapse of the fixed exchange rate.

Using a sample model, it can be shown that an increase in initial reserves or a decrease in domestic credit growth or both delay the collapse of fixed rate. When government balances its budget or stops financing deficit by increasing money supply, domestic credit growth is reduced to zero and crisis can be avoided. All of these imply that a currency crisis is predictable, both for its occurrence and timing in first-generation model.

2.2. Self-fulfilling Models

[3] Paul Krugman, "O Canada: A neglected nation gets its Nobel", Slate Magazine, October 19, 1999.

The first-generation model did work quite well for early crises in 1980s when most crises came with irresponsible monetary policy. However, problem came when United Kingdom was attacked and exited European Monetary System without significant inconsistent monetary policies. Interest rate was first raised to save the link but then put away when it became too high, which suggested that United Kingdom chose, instead of was forced, to exit from the system. Under such backdrop, the second-generation came to researchers' attention.

The second-generation model, originated in Obstfeld (1986) and further illustrated in Obstfeld (1994, 1996) argues that currency crises are self-fulfilled. In self-fulfilling model, inadequate economic fundamentals are no more a necessary condition for currency crises.

Sufficient reserve endorses government's ability to defend its fixed exchange rate, but not its willingness. In the first-generation model, government is forced to abandon fixed exchange rate when foreign currency reserve is exhausted without any alternatives. In the second-generation model, on the other hand, abandonment of fixed exchange rate is government's voluntary action to minimize social loss. Government faces different objectives, including stable exchange rate, inflation, employment, etc. A fixed exchange rate reduces social loss by achieving stable exchange rate and price level, but loses the opportunity to optimize monetary policy to achieve full employment or stimulate output. If difference between actual and expected change in exchange rate is positively related to output level[4] (i.e. output increases when actual depreciation is larger than expected depreciation), an increase

[4] Self-fulfilled currency crisis can be illustrated with many other social loss functions. The one used in the paper is captured from Obstfeld (1996).

in depreciation expectation causes an increase in social loss by reducing output under fixed exchange rate. Increasing depreciation expectation causes increasing social loss of using fixed exchange rate and government chooses to abandon fixed exchange rate to minimize social loss at certain threshold. Therefore, expectation is realized because of itself or self-fulfilled[5].

Based on the social loss framework which abandonment of fixed exchange rate policy is a choice, Chan and Chen (2003) suggested a central bank can use an irrevocable commitment policy to defend its fixed exchange rate[6]. The idea is simple and straight forward, irrevocable commitment increases the abandonment cost of the fixed exchange rate (or increases the social loss of flexible exchange rate in a more direct way) and thus lowers the expected probability of a currency crisis.

2.3. Contagious Currency Crises

Not surprisingly, currency crises are usually contagious. Every time a currency crisis happens in one country, it spreads to other countries in the region or with similar economic structure. In the Asian crisis, currencies of other countries in the region such as Malaysia, Philippines collapsed shortly after the fall of Thai baht.

Kasa and Huh (2001) argued that contagious currency crises were caused by export competition. Contagious currency crises or competitive devaluations are again, a voluntarily loss minimization action of countries. In the model, social loss

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[5] Self-fulfilling model could be more complicated with multi-equilibria for depreciation expectation. So, even a sunspot could shift expectation from one equilibrium to another which cause a change of exchange rate policy.

[6] The original exchange rate regime used by Chan and Chen (2003) was currency board. However, the idea is also applicable on other hard-fixed exchange rate systems.

is composed of a deviation of output from "natural" level and price level. The actual output depends on relative exchange rate, that is the difference between exchange rate of a country and the group average exchange rate, and a common shock to the region. The relative exchange rate in the social loss function makes currency crisis contagious. Finally, present value of the social loss equals to discounted social loss for all periods.

If central banks can cooperate and commit to an exchange rate policy, the joint loss minimizing strategy is setting exchange rates at zero for all countries. The one-period social loss for a country is given as L^C. Similar to monetary authority's temptation to increase money supply once inflation expectation is formed, a country also temps to deviate from the average exchange rate. With the optimized exchange rate (when exchange rates of other countries is fixed at zero), the one-period social loss is given as L^D. The benefit from deviation is therefore L^C - L^D. Benefit of deviation increases with number of countries in the group (i.e. can seize export from a larger number of countries). Thus, cooperation among a larger group of countries is more difficult to sustain.

Once a country devalues its currency, other countries in the group are going to response by devaluing theirs in second period. Finally, all countries reach another equilibrium with all exchange rates being equal and social loss becomes L^N. Therefore, a country suffers an increase in social loss from competitive devaluation which equals to $L^N - L^C$ in long run.

Decision to deviate from the committed exchange rate or not is a comparison between its benefit in the first period (L^C - L^D) and costs. Such costs include present value of 1) loss from competitive devaluation ($L^N - L^C$) if deviation is not required in future and 2) loss from loss reduction opportunity (L^C - L^D) if deviation is required in future. Based on the benefit and costs, a critical level of negative shocks (u*) for devaluation can be found. When negative shocks exceed this critical level, there will be simultaneous devaluations for countries in the group .

3. The Model

3.1. Output Stability and Price-level Stability Tradeoff

The basic setup of the model basically follows the framework of Kasa and Huh (2001). Two central banks, facing symmetric tradeoff between output stability and price-level stability. In other words, each central bank has following social loss function:

$$L_i = \frac{1}{2}(y_i - \bar{y})^2 + \frac{\varepsilon}{2} p_i^2, \tag{1}$$

where y_i denotes the log of country *i*'s output, $\bar{y}$ is the central bank's target level of output and ε is the central bank's relative preference on price-level stability and output stability. The larger the ε, the larger the social loss assigned by the central bank on price level variability relative to output variability. Inflation rate of country *i* is denoted by p_i.

An important assumption in the framework of Kasa and Huh (2001) is that countries compete for export sales to a common external market. As a result of export competition, export of country *i* are an increasing function of its depreciation rate relative to another country (denoted by *j*). This relationship is captured by the following output function[7]:

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[7] The function implies that output of country i's output equals to zero when $e_i = e_j$. It may make not much sense. Such problem can be solved by including a constant term, for example y_0, in the output function like the one used in Obstfeld (1996), without affecting the findings of this paper. Existing output is used for simplicity.

$$y_i = \alpha(e_i - e_j), \tag{2}$$

where e_i and e_j denotes the depreciation rate of country i and j respectively while α denotes the sensitivity of country i's output to the relative depreciation rate.

The relationship between domestic inflation rate and the depreciation rate is assumed to be deterministic as mentioned in Kasa and Huh (2001):

$$p_i = \beta e_i, \tag{3}$$

where β is the sensitivity of country i's inflation rate to the depreciation rate of its currency. Such partial pass-through, as explained in Kasa and Huh (2001), can come from an upward-sloping aggregate supply curve or a domestic price index with imported goods[8].

Substituting equation (2) and (3) into (1) gives the following social loss function:

$$L_i = \frac{1}{2}\left(\alpha\left(e_i - e_j\right) - \bar{y}\right)^2 + \frac{\gamma}{2}e_i^2, \tag{4}$$

where γ is now the central bank's inflation-output stability preference adjusted by the degree of pass-through, that is $\gamma = \beta^2\varepsilon$.

[8] Please see Appendix 8.1 for details.

In a symmetric equilibrium where $e_i = e_j$, putting (3) and (4) into (1) and the social loss function becomes

$$L_i = \frac{1}{2}\bar{y}^2 + \frac{\gamma}{2}e_i^2. \tag{5}$$

It is clear that the joint loss-minimizing policy is to set $e_i = e_j = 0$ or in other words, adopt a fixed exchange policy if two central banks can commit to an exchange rate policy. Thus, the loss for this cooperative equilibrium is as below:

$$L_i^{ff} = \frac{1}{2}\bar{y}^2. \tag{6}$$

Central bank which adopts fixed exchange rate policy, however, forgoes the opportunity to minimize its social loss through optimizing its depreciation rate (see details in the Appendix) as below:

$$e_i = \frac{\alpha\bar{y} + \alpha^2 e_j}{\alpha^2 + \gamma}. \tag{7}$$

If a central bank chooses to depreciate its currency unilaterally, it enjoys a lower loss than the cooperative equilibrium (see details in the Appendix):

$$L_i^{lf} = \frac{1}{2}\bar{y}^2\left(\frac{\gamma}{\alpha^2 + \gamma}\right), \tag{8}$$

while another central bank keeping the fixed exchange rate policy suffers a higher social loss (see details in the Appendix):

$$L_i^{fl} = \frac{1}{2}\bar{y}^2\left(1 + \frac{\alpha^2}{\alpha^2 + \gamma}\right)^2. \tag{9}$$

Temptation of a unilateral devaluation may make two central banks fail to cooperate when both banks try to stimulate export sales with flexible exchange rate policy. The reaction function in (7) means central banks will fall into competitive devaluations under flexible exchange rate policy, each central bank chooses a higher depreciation rate than its competitor's. At equilibrium where $e_i = e_j$, the reaction function in (7) provides following equilibrium depreciation rate under competitive devaluations (see details in the Appendix):

$$e_i = e_j = \frac{\alpha}{\gamma}\bar{y}, \tag{10}$$

resulting in following social loss (see details in the Appendix)

$$L_i^{ll} = \frac{1}{2}\bar{y}^2\left(1 + \frac{\alpha^2}{\gamma}\right). \tag{11}$$

As a short summary, flexible exchange rate policy gives central bank a tool to minimize social loss by optimizing depreciation rate. Unfortunately, if both central banks adopt flexible exchange rate policy to obtain the optimization benefit, each central bank will keep responding to another central bank's depreciation. Two central banks will finally fall into the competitive devaluations equilibrium and both suffer from a social loss higher than the cooperative equilibrium.

3.2. Realignment Cost

Social loss function introduced in previous section mainly focuses the tradeoff between output stability and price-level stability as described in the framework of Kasa and Huh (2001). Models in other literatures, however, introduce addition costs involved in different exchange rate policy other than social loss from output variability and price-level variability. Fixed costs involved in adopting the flexible exchange rate policy were introduced in various currency crises models such as Obstfled (1996), Chan and Chen (2003), etc. Such fixed costs can be due to an increase in exchange rate volatility, loss of credibility on previous fixed exchange rate commitment, etc. The fixed costs are summarized as a country-specific realignment cost, C_i in this model. The realignment cost, C_i is incorporated in the social loss function for unilaterally devaluation, (8) and competitive devaluations, (11) as below:

$$L_i^{lf} = \frac{1}{2}\bar{y}^2\left(\frac{\gamma}{\alpha^2+\gamma}\right)+C_i, \tag{12}$$

$$L_i^{ll} = \frac{1}{2}\bar{y}^2\left(1+\frac{\alpha^2}{\gamma}\right)+C_i. \tag{13}$$

3.3. Speculative Attack and Its Size

In an economy without capital control, a central bank with fixed exchange rate policy is obligated to convert domestic currency into foreign currency upon request at the specified exchange rate. In this sense, a speculative attack on the fixed exchange rate results in a significant outflow of foreign reserves and become an additional cost in defending the fixed exchange rate policy. In Morris and Shin (1998) and Angeletos et al (2006), the cost of defending a fixed exchange rate policy is an increasing function of the size of speculative attack.

Speculators are motivated to attack a fixed exchange rate by the potential return of its abandonment. In a typical speculative attack, speculators short sell the currency at the official fixed exchange rate and will make a profit by settling the short position with the depreciated currency when they successfully force the central bank to abandon the fixed exchange rate policy. Obviously, the expected depreciation rate (Ee_i) is a key determinant of speculators' return and thus, the size of speculative attack,

$$S_i = S(Ee_i)\,. \qquad (14)$$

The next question is how the expected depreciation rate is determined. The expected depreciation rate should be a product of the probability of a successful attack (i.e. central bank abandons the fixed exchange rate policy) and the depreciation rate of the currency once the fixed exchange rate policy is abandoned. Both factors are different under different foreign exchange rate policy (i.e. the exchange rate policy of the export competing country). To determine the probability of a successful speculative attack, it is assumed that the actual aggregate size of the speculative attack is unknown to individual speculator but is believed to be uniformly distributed on $[0,\bar{S}]$. Individual speculator depends on the expected depreciation rate to make a one-off decision[9] to determine their individual attack size. Depreciation rate of the currency after a successful speculative attack, on the other hand, is determined by (7). Details of the determinants of expected depreciation rate under different foreign exchange rate policy are summarized in the following table.

Foreign central bank's policy	$\Pr(S > \hat{S})$	$E(e \mid S > \hat{S})$
Fixed	$1-\frac{1}{\bar{S}}\left(C_i - \lambda^f\right)$	$\left(\frac{\alpha}{\alpha^2+\gamma}\right)\bar{y}$
Flexible	$1-\frac{1}{\bar{S}}\left(C_i - \lambda^l\right)$	$\left(\frac{\alpha}{\gamma}\right)\bar{y}$

Table 1 – Expected depreciation rate under different foreign exchange rate policies[10]

Costs related to a speculative attack on the fixed exchange rate are incorporated in (6) and (9) as below:

$$L_i^{ff} = \frac{1}{2}\bar{y}^2 + S_i^f, \tag{15}$$

$$L_i^{fl} = \frac{1}{2}\bar{y}^2\left(1+\frac{\alpha^2}{\alpha^2+\gamma}\right)^2 + S_i^l, \tag{16}$$

[9] Otherwise, size of the speculative attack can be determined with an unknown realignment costs (C) to speculators. Current form of speculative attack size determination is used for simplicity.

[10] Please refer λ^f and λ^l to (17) and (19) respectively

where S_i^f and S_i^l are the size of speculative attack on a central bank when another central bank adopts fixed and flexible exchange rate policy respectively. Further discussion of S_i^f and S_i^l as well as its implications will come shortly in the next section.

4. Two-Stage Game for Exchange Rate Policy Decision

In most currency crises, for example, the recent Asian Financial Crisis, the story starts with a group of countries all adopted fixed exchange rate policy. When one of the countries was forced to abandon its fixed exchange rate policy by speculators, some countries followed the abandonment decision shortly while other continued to maintain the fixed exchange rate policy. A scenario of simultaneous devaluations illustrated in Kasa and Huh (2001) seldom happened in the history.

Before talking about the financial aid, exchange rate policy interaction of two central banks will be studied first. The basic model setup introduced in the last section is incorporated into a two-stage game to see how a foreign and a domestic central bank will formulate their exchange rate policy in such game.

4.1. The Game

Consider a two-stage game for exchange rate policy formulation. In the first stage, foreign central bank decides its exchange rate policy and the domestic central bank responses to the foreign central bank's policy in the second stage. The game is captured in the following game tree.

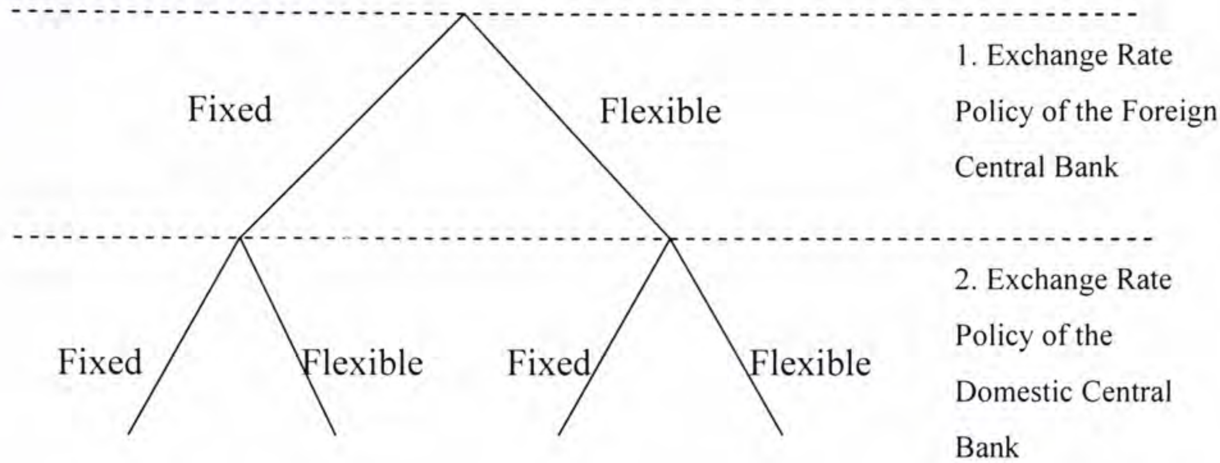

Figure 1 - Two-country game for exchange rate policy decision

The story begins with a cooperative equilibrium where two central banks both adopt the fixed exchange rate policy, without any speculative attack ($S=0$). From (12) and (15), a necessary condition for domestic central bank to attain the cooperative equilibrium is that its realignment cost must be larger than the optimization benefit under a foreign fixed exchange rate policy, λ^f (i.e. $L^{ff} < L^{lf}$).

$$C_{Domestic} > \lambda^f \qquad \text{where } \lambda^f = \frac{1}{2}\bar{y}^2\left(\frac{\alpha^2}{\alpha^2+\gamma}\right). \tag{17}$$

4.1.1. Policy Response of the Domestic Central Bank

Next, speculators attack the fixed exchange rate policy of the foreign central bank. The foreign central bank is forced to abandon its fixed exchange rate policy when the size of speculative attack is large enough to make the foreign central bank involves larger social loss in maintaining the fixed exchange rate policy than abandoning it. Response of the domestic central bank to the change of the foreign central bank's exchange rate policy depends on the domestic central bank's realignment cost. Speculative attack can be repeated, which means the domestic central bank's fixed exchange rate can become speculators' next target. From (13) and (16), the domestic central bank will maintain its fixed exchange rate policy only if its realignment cost is sufficient large to offset the speculative attack pressure and optimization benefit, given with a foreign flexible exchange rate policy (i.e. S_D^l and λ^l). Otherwise, the domestic central bank will choose to follow the foreign central bank to abandon the fixed exchange rate and a contagious currency crisis will be observed in such case. Policy responses of the domestic central bank to a change of foreign exchange rate policy are summarized as follow:

$$\text{Domestic policy choose}\begin{cases} \text{flexible exchnage rate policy} & (\text{for } C_{Domestic} < \lambda^l + S_D^l) \\ \text{fixed exchange rate policy} & (\text{for } C_{Domestic} \geq \lambda^l + S_D^l) \end{cases}, \tag{18}$$

where

$$\lambda^l = \frac{1}{2}\bar{y}^2\left[\left(1+\frac{\alpha^2}{\alpha^2+\gamma}\right)^2 - \left(1+\frac{\alpha^2}{\gamma}\right)\right]. \tag{19}$$

4.1.2. Policy Decision of the Foreign Central Bank

Policy response of the domestic central bank in (18) in turn affects the exchange rate policy decision of the foreign central bank. In the two-stage game illustrated in Figure 1, social loss of the foreign central bank to abandon the fixed exchange rate policy depends on the domestic central bank's policy response. If the domestic central bank will keep its fixed exchange rate policy anyway, the social loss of the foreign central bank will be (12). If the foreign central bank's abandonment will be followed by the domestic central bank, the social loss of the foreign central bank will be (13). The minimum size of speculative attack which is required to make the foreign central bank to deviate from the cooperative equilibrium (denoted by $\hat{s}_F^f$) under different domestic central bank's policy responses can be obtained from the social loss difference between (15) and (12), (15) and (13) respectively. The amount of $\hat{s}_F^f$ is captured in (20). Clearly, $\hat{s}_F^f$ is higher when the domestic central bank threatens to use the competitive devaluations strategy (i.e. abandoning the fixed exchange rate policy when the foreign central bank abandons its fixed exchange rate policy).

$$\hat{s}_F^f = \begin{cases} C_{Foreign} + \dfrac{\alpha^2}{2\gamma}\bar{y}^2 & (\text{for } C_{Domestic} < \lambda^l + s_D^l) \\ C_{Foreign} - \lambda^f & (\text{for } C_{Domestic} > \lambda^l + s_D^l) \end{cases}. \tag{20}$$

4.2. Special Features of the Game

The model has two important features. Firstly, impact of the output sensitivity to the depreciation rate (α) and the adjusted inflation-output stability preference (γ) on the domestic central bank's policy response is non-monotonic. Such feature implies that collapse of one country's fixed exchange rate is more likely to result in competitive devaluations with intermediate parameters. Secondly, a central bank which is able to defend its fixed exchange rate policy from speculative attack at the cooperative equilibrium may fail to do so when speculators first attack the "weakest link".

4.2.1. Export Sensitivity, Adjusted Inflation-Output Stability Preference and Policy Response

From (18) and (19), it is not difficult to see that α and γ are important determinants of the domestic central bank's policy response through their impacts on the optimization benefit (λ^l) and the size of speculative attack (s_D^l). To analyze their relationship with policy response of the domestic central bank, let α^2/γ be φ and thus λ^l described in (19) can be rewritten as:

$$\lambda^l = \frac{1}{2}\bar{y}^2\left[\left(\frac{1+2\varphi}{1+\varphi}\right)^2 - (1+\varphi)\right], \tag{21}$$

where $0 \leq \varphi \leq \infty$.

Intuitively, social loss of the domestic central bank at competitive devaluations (the second part of (21) or $0.5\bar{y}^2(1+\varphi)$) increases proportionally with φ while social loss of maintaining a fixed exchange rate under a foreign flexible exchange rate policy (the first part of (21)) increases with a decreasing rate. Analytical study shows that the optimization benefit maximize when φ equals to 0.675 (see details in the Appendix). Relationship between λ^l and φ is presented graphically in Figure 4 which clearly shows that λ^l is a non-monotonic function of φ.

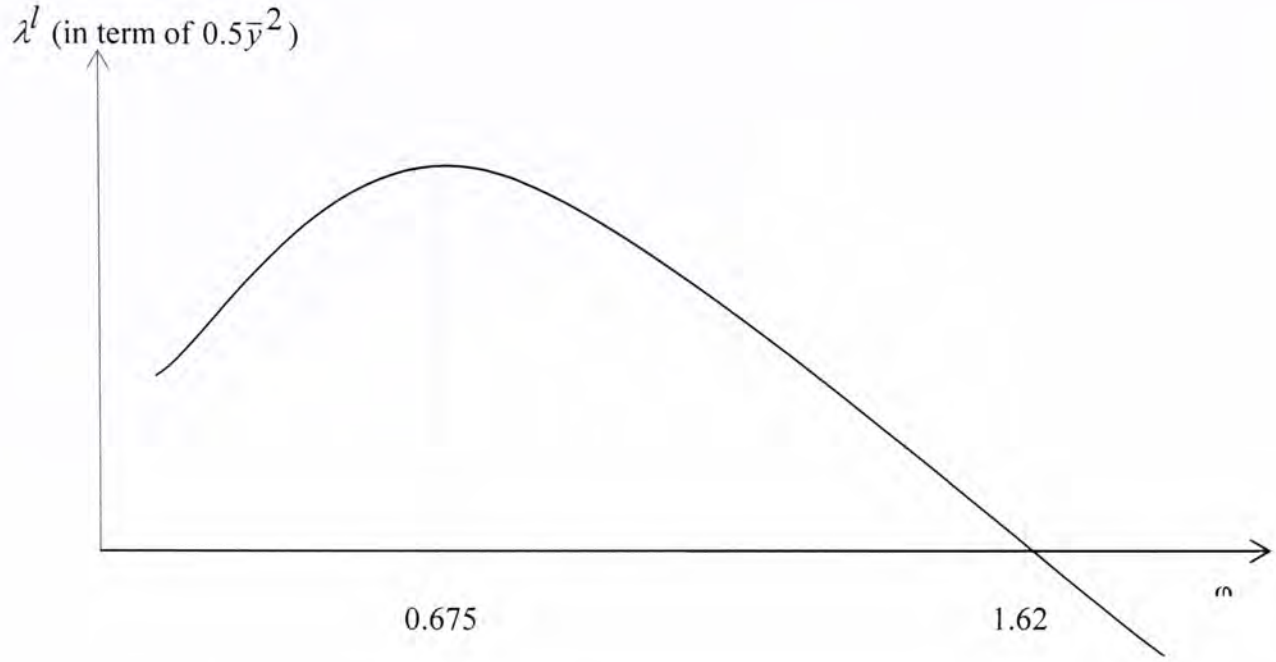

Figure 2 - Relationship between λ^l and φ

Turning to the size of speculative attack (S_D^l), Table 1 shows that probability of de-link ($\Pr(S > \hat{S})$) depends directly on the optimization benefit (λ^l). So, non-monotonic relationship between λ^l and φ also implies that the probability of de-link is a non-monotonic function of φ. The expected depreciation rate under foreign flexible policy ($E(e \mid S > \hat{S})$), on the other hand, is simply an increasing function of φ. Thus, it is possible to exist multiple critical values for competitive devaluations (i.e. $\lambda^l + S_D^l$).

An example for multiple critical values for competitive devaluations is given in Figure 3. According to the numerical example, the domestic central bank will maintain its fixed exchange rate policy when $\varphi \leq 1.04$ and $\varphi \geq 14.65$ while it will follow the foreign central bank to abandon the fixed exchange rate policy when $1.04 < \varphi < 14.65$. Intuitively, a low output sensitivity makes the domestic central bank suffers less from the abandonment of the foreign central bank's fixed exchange rate policy such reduces the benefit of competitive devaluations. On the other hand, a high output sensitivity results in significant increase in the social loss of competitive devaluations. Therefore, the domestic central bank will prefer to maintain its fixed exchange rate policy when the foreign fixed exchange rate falls due to the threat of competitive devaluations.

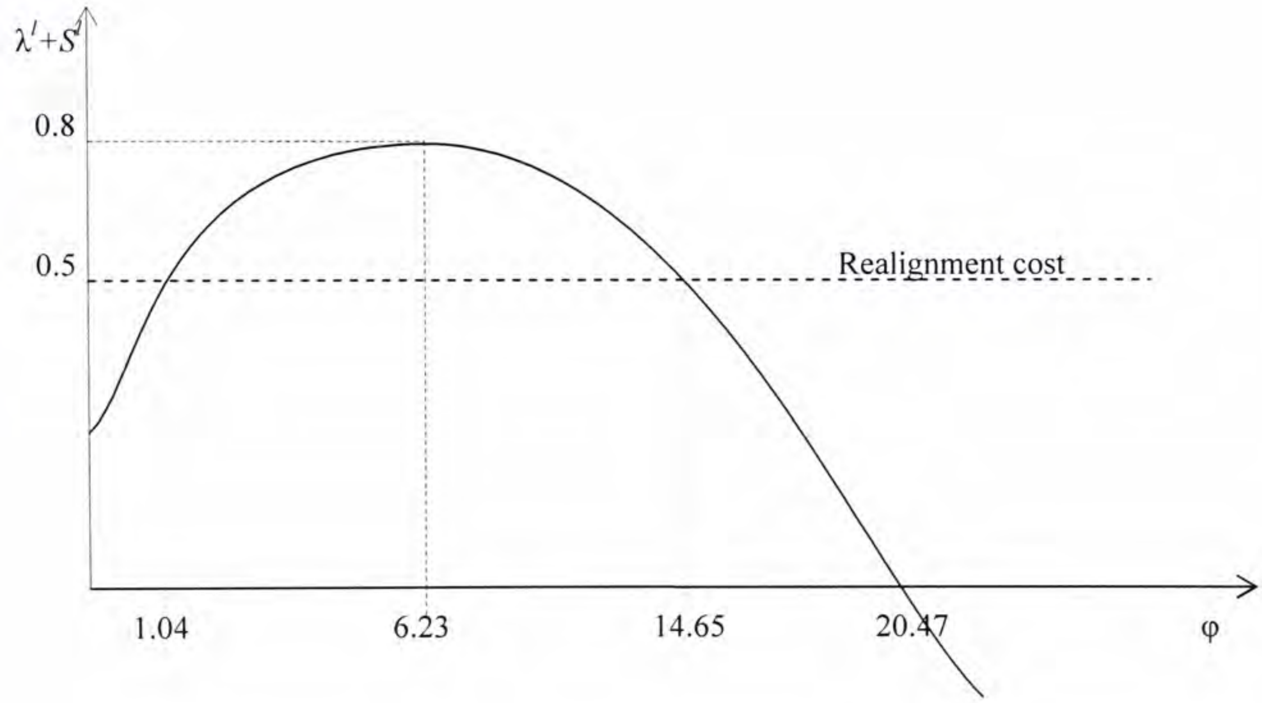

Figure 3 - An example for multiple critical levels for exchange rate policy response

$\bar{y}^2 = 0.1, \bar{S} = 2, S = 2 * Ee$

4.2.2. Speculative Attack through the "Weakest Link"

According to the model, a central bank which cannot be forced to abandon its fixed exchange rate policy by speculators directly can be forced to do so after speculators successfully forced another currency to de-link. At the cooperative equilibrium, the minimum size of speculative attack required to force the domestic central bank to deviate from the cooperative equilibrium (denoted by $\hat{s}^f_D$) is:

$$\hat{s}^f_D = C_{Domestic} - \lambda^f \tag{22}$$

Once the foreign central bank floats its currency, the minimum size of speculative attack which is required to make domestic central bank to abandon its fixed exchange rate policy (denoted by $\hat{s}^l_D$) is:

$$\hat{s}^l_D = C_{Domestic} - \lambda^l \tag{23}$$

Note that the optimization benefit is always larger when foreign exchange rate policy is fixed (i.e. $\lambda^f > \lambda^l$) (see details in the Appendix). Such a result comes from the threat of competitive devaluation which increases a central bank's social loss significantly. Thus, the optimization benefit under foreign central bank's flexible exchange rate policy reduces and even turns into negative. From (22) and (23), $\lambda^f > \lambda^l$ implies that $\hat{s}^l_D > \hat{s}^f_D$. Therefore, a constant size of speculative attack (i.e. $s^l_D = s^f_D$) means a central bank that can defend its fixed exchange rate policy against speculative attack under a foreign fixed exchange rate policy must be able to defend it under a foreign flexible exchange rate policy.

From Table 1, the probability of de-link under a foreign fixed exchange rate policy is always higher than or equal to the one under foreign fixed policy as

$\hat{s}_D^l > \hat{s}_D^f$. The expected depreciation rate once the currency is de-linked, however, is higher when the foreign exchange rate policy is flexible due to competitive devaluations. The higher expected depreciation rate in competitive devaluations therefore may overweight the lower de-link probability and result in a higher *Ee* and thus, a greater size of speculative attack on the domestic central bank under foreign flexible exchange rate policy. It means that it is possible to have $s_D^l > \hat{s}_D^l > \hat{s}_D^f > s_D^f$. In other words, a central bank which can not be forced by speculative attack to de-link its currency at cooperative equilibrium may be forced to do so after fixed exchange rate of its "weaker" export competing central bank (i.e. export competing central bank with lower realignment cost) collapsed.

5. Financial Aid in Currency Crisis

With the basic setup of the model and the introduction of a simple game for exchange rate policy interaction, this section moves forward to discuss financial aid. For simplicity, this section focuses on the relationship between financial aid and realignment cost by keeping the output sensitivity and the adjusted inflation-output stability preference constant.

5.1. The Game with Financial Aid

Assume that two central banks are able to stay at the cooperative equilibrium when there is no speculative attack. The domestic central bank has a relative high realignment cost and speculators are unable to force it to abandon its fixed exchange rate policy from the cooperative equilibrium. The foreign central bank, on the other hand, has a relative low realignment cost and will choose to change its exchange rate policy from fixed to flexible under speculative attack (denoted by S_F^f). It is shown in (21) that the minimum size of speculative attack required for the foreign central bank to deviate from the cooperative is larger under the threat of competitive devaluations. To allow different exchange rate policy responses in the study, the size of speculative attack is assumed to be sufficient large to force foreign central bank to deviate from the cooperative equilibrium regardless of the domestic central bank's response. Abovementioned information is summarized in the following assumptions.

ASSUMPTION A1. $C_{Domestic} > S_D^f + \lambda^f > \lambda^f$

ASSUMPTION A2. $S_F^f - 0.5\bar{y}^2(\alpha^2 / \gamma) > C_{Foreign} > \lambda^f$

The abandonment of foreign central bank's fixed exchange rate policy always results in a higher loss for the domestic central bank and therefore the domestic central bank has a motivation to intervene the foreign central bank's choice of exchange rate policy.

Lowering the foreign central bank's loss in maintaining fixed exchange rate policy by providing financial aid is a possible way. For simplicity, no repayment is required for the financial aid, so future value of the repayment can be ignored. The financial aid is only provided to the foreign central bank if it agrees to maintain its fixed exchange rate policy and the domestic central bank can withdraw the financial aid any time if the foreign central bank fails to do so. The withdrawal option is important because if financial aid can not be withdrawn once provided, the financial aid becomes irrelevant for the foreign bank in deciding its exchange rate policy. After foreign central bank decides its foreign exchange rate policy with the optional financial aid, domestic bank decides its own policy response. Speculators can shift its target from foreign central bank to domestic central bank at any time. So, domestic central bank will consider the presence of speculative attack in deciding its exchange rate policy response. Under A1 and A2, the game for the currency crisis with financial aid is illustrated in Figure 4[11].

[11] Please see Appendix 8.8 for the complete two-country game tree.

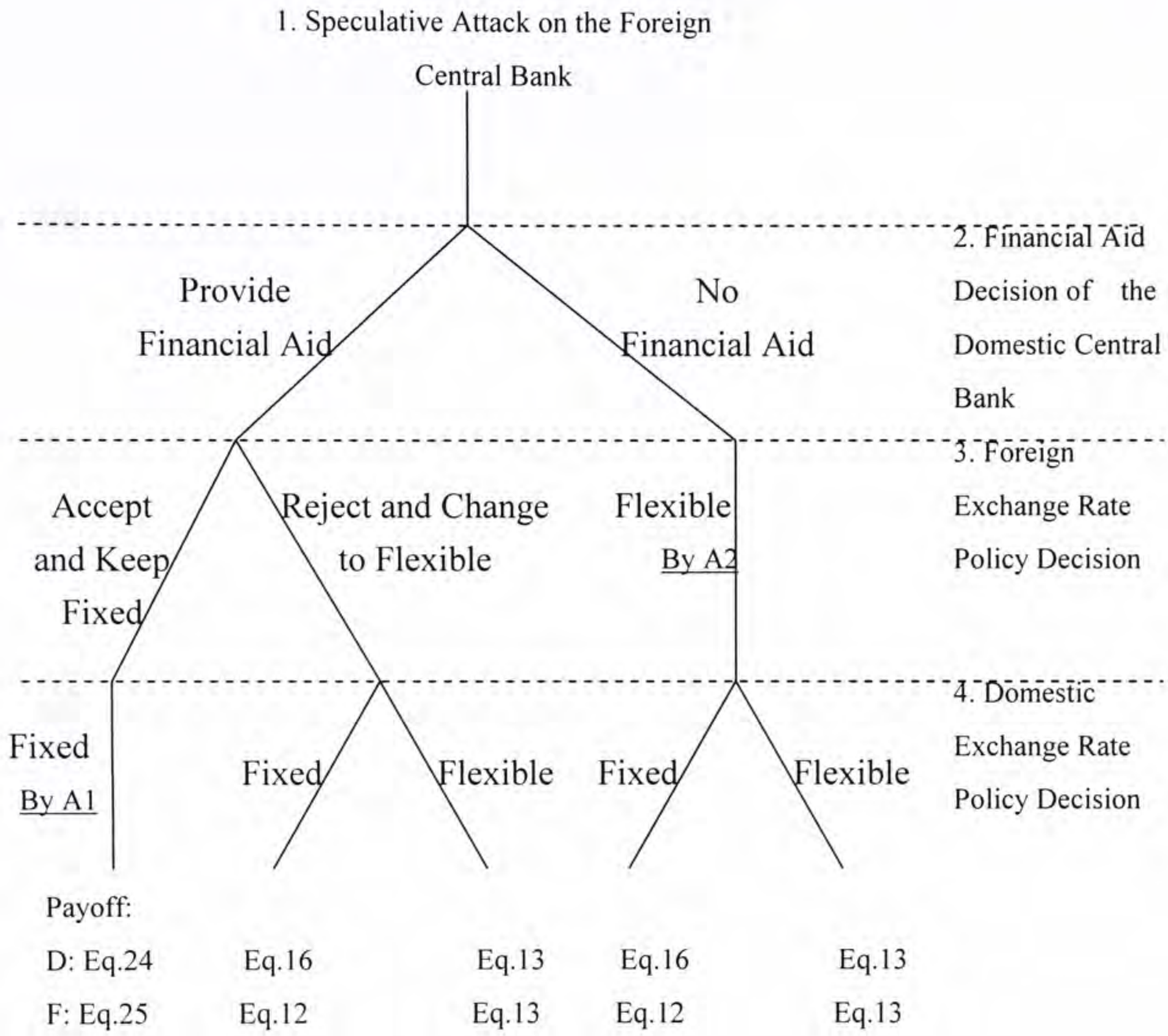

Figure 4 - Two-country game with financial aid

5.2. Stage 4: Policy Response of the Domestic Central Bank

In response to the change of the foreign exchange rate policy from fixed to flexible, policy choice of the domestic central bank is captured in (18) depends on realignment cost of the domestic central bank. If the foreign central bank accepts the financial aid and maintain its fixed exchange rate policy, the domestic central bank must keep its own fixed exchange rate policy according to Assumption 1.

Even though the financial aid helps reducing social loss of the foreign central bank at cooperative equilibrium, it costs the domestic central bank. The financial aid is incorporated in the social loss functions of the foreign and domestic central bank as below:

$$L^{ff}_{Domestic} = \frac{1}{2}\bar{y}^2 + S^f_D + F\,, \tag{24}$$

$$L^{ff}_{Foreign} = \frac{1}{2}\bar{y}^2 + S^f_F - F\,, \tag{25}$$

where F is the log of financial aid.

5.3. Stage 3: Policy Decision of the Foreign Central Bank

If the domestic central bank decides not to provide financial aid or the foreign central bank rejects the financial aid provided by the domestic central bank, the foreign central bank must abandon its fixed exchange rate according to Assumption 2. Social loss of the foreign central bank will then depends on the domestic central bank's policy response. Associate (12) and (13) with the domestic central bank's policy responses stated out in (18), social loss of the foreign central bank in changing its exchange rate policy is as below

$$\begin{array}{l} \text{Social loss of the} \\ \text{Foreign Central Bank} \end{array} \begin{cases} C_{Foreign} + \frac{1}{2}\bar{y}^2\left(1+\frac{\alpha^2}{\gamma}\right) & (\text{for } C_{Domestic} < \lambda^l + S^l_D) \\ C_{Foreign} + \frac{1}{2}\bar{y}^2\left(\frac{\gamma}{\alpha^2+\gamma}\right) & (\text{for } C_{Domestic} \geq \lambda^l + S^l_D) \end{cases} \tag{26}$$

On the other hand, the foreign central bank can accept the financial aid provided by the domestic central and stay on the cooperative equilibrium. Social loss of the foreign central bank at cooperative equilibrium with compensation from the financial aid is as (25). The foreign central bank will only accept the financial aid when its loss at the compensated cooperative equilibrium is lower than the one in (26), depending on realignment cost of the domestic central bank. Comparing social loss of the foreign central bank in (25) and (26), the minimum financial aid required to keep foreign central bank at the cooperative equilibrium, F_{min} is

$$F_{\min}\begin{cases} S^{f} - C_{Foreign} - \frac{1}{2}\bar{y}^{2}\left(\frac{\alpha^{2}}{\gamma}\right) & (\text{for } C_{Domestic} < \lambda^{l} + S_{\mathrm{D}}^{l}) \\ S^{f} - C_{Foreign} + \frac{1}{2}\bar{y}^{2}\left(\frac{\alpha^{2}}{\alpha^{2}+\gamma}\right) & (\text{for } C_{Domestic} \geq \lambda^{l} + S_{\mathrm{D}}^{l}) \end{cases} . \qquad (27)$$

Clearly, the threat of competitive devaluations reduces the foreign central bank's optimization benefit and thus reduces the minimum financial aid required to keep the foreign central bank at the cooperative equilibrium.

5.4. Stage 2: Financial Aid Decision of the Domestic Central Bank

Unless the domestic central bank provides the minimum financial aid in (27), the foreign central bank will reject the offer of financial aid from the domestic central bank. In this case, the domestic central bank is indifference between providing "insufficient" financial aid and no financial aid. Associate (13) and (16) with the domestic central bank's policy response in (18), social loss of the domestic central bank under different scenarios is as below:

$$
\text{Social loss of the Domestic Central Bank}\begin{cases} C_{Domestic}+\frac{1}{2}\bar{y}^2\left(1+\frac{\alpha^2}{\gamma}\right) & (\text{for } C_{Domestic}<\lambda^l+S_{\mathrm{D}}^l) \\ S_D^l+\frac{1}{2}\bar{y}^2\left(1+\frac{\alpha^2}{\alpha^2+\gamma}\right)^2 & (\text{for } C_{Domestic}\geq\lambda^l+S_{\mathrm{D}}^l) \end{cases}. \tag{28}
$$

Although the cooperative equilibrium can be maintained by providing a financial aid, the domestic central bank will simply forgo this option if the F_{min} makes social loss in (24) higher than (28). The maximum financial aid that the domestic central bank is willing to provide, F_{max} is obtained from the difference between (24) and (28):

$$
F_{\max}\begin{cases} C_{Domestic}-S_D^f+\frac{1}{2}\bar{y}^2\left(\frac{\alpha^2}{\gamma}\right) & (\text{for } C_{Domestic}<\lambda^l+S_D^l) \\ S_D^l-S_D^f+\frac{1}{2}\bar{y}^2\left[\left(1+\frac{\alpha^2}{\alpha^2+\gamma}\right)^2-1\right] & (\text{for } C_{Domestic}\geq\lambda^l+S_D^l) \end{cases}. \tag{29}
$$

The maximum financial aid from the domestic central bank is first an increasing function of the realignment cost when realignment cost is low and the fixed exchange rate policy is not a dominant strategy. At this region, higher realignment cost increases the domestic social loss at competitive devaluations while decreases the speculative pressure at the original cooperative equilibrium. Once realignment cost increases to a critical level which makes fixed exchange rate policy a dominant strategy, further increase in the realignment cost continues to reduce speculative pressure on the domestic central bank. However, the domestic speculative pressure decreases at a faster rate when the foreign central bank adopts a flexible rate policy and thus the maximum financial aid becomes a decreasing function of the realignment cost. Finally, the realignment cost increases to a level which makes a de-peg decision impossible[12] and maximum financial aid will rest on a stable level. Such a pattern has an interesting intuition: financial aid may rescue the cooperative equilibrium with intermediate domestic realignment cost, but fails to do so with either low or high domestic realignment cost. Such case is illustrated in Figure 5.

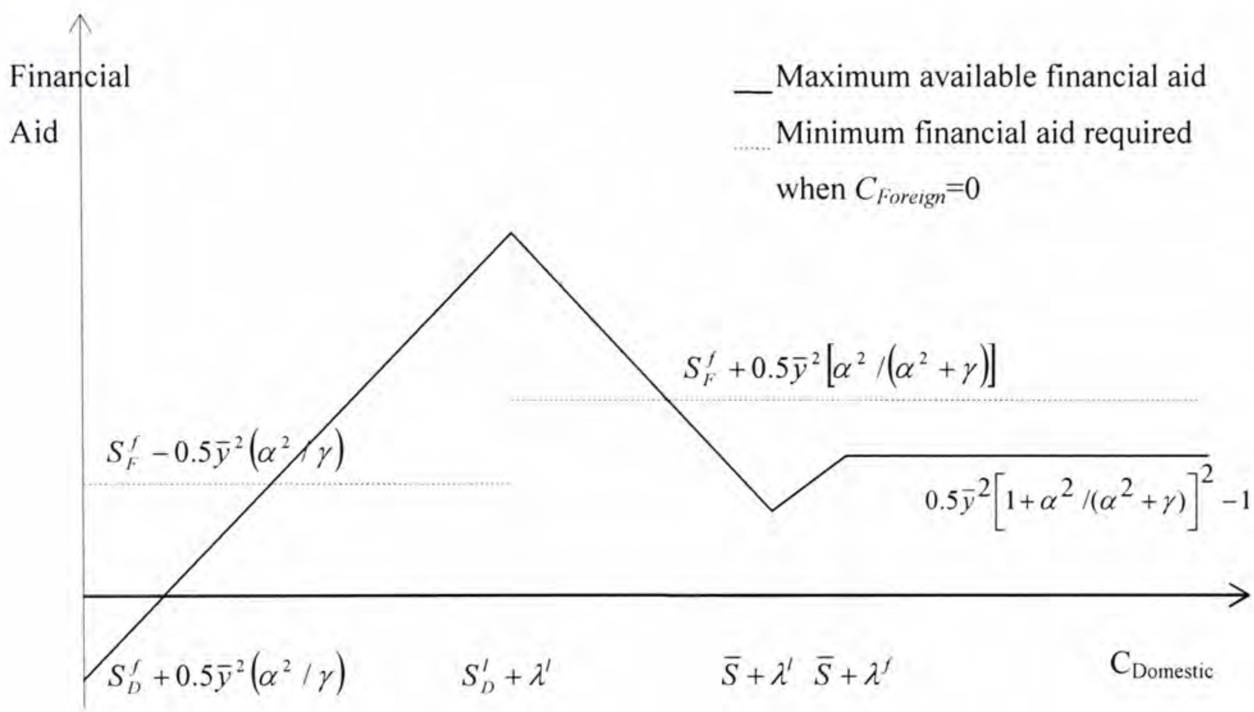

Figure 5 - An illustration for financial aid.
$\alpha^2 = 2, \gamma = 1, \bar{y}^2 = 0.1, \bar{S} = 2, S = 2 * Ee$

From the illustrative example, the region with negative financial aid from the domestic central bank is actually avoided by the Assumption 1. At the region where $C_{Domestic} < \lambda^l + S_D^l$, the domestic central bank will follow the foreign central bank to abandon the fixed exchange rate policy and minimum financial aid required by the foreign central bank is lower under the threat of competitive devaluations. Since the domestic central bank needs to abandon its fixed exchange rate policy,

[12] It means that $\Pr(S > \hat{S})$ under both foreign fixed and flexible exchange rate policy equal to zero.

increasing realignment cost keeps motivating the domestic central bank to provide more financial aid at this region. However, once $C_{Domestic} \geq \lambda^{l} + S_{D}^{l}$, realignment cost of the domestic central bank is high enough to make fixed exchange rate policy a dominant strategy of the domestic central bank. Removing the threat of competitive devaluations, there is a jump in the minimum financial aid requested by the foreign central bank. The maximum financial aid that will be provided by the domestic central bank, however, start decreasing with the increasing domestic realignment cost and finally below the level required to keep the foreign central bank at the cooperative equilibrium. The illustrative example shows that an intermediate instead of high domestic realignment cost helps to sustain a cooperative equilibrium through financial aid.

6. Concluding Remarks

Existing studies mainly focus on the cause of currency crises but seldom take financial aid into consideration. Interrelationship between the exchange rate policies of central banks described in Kasa and Huh (2001) is only the first half of the story. As long as negative impact of another currency's devaluation on the social loss of domestic economy is noted, central banks must try to avoid it to obtain constrained maximization. Financial aid is one of the solutions.

Actually, financial aid is an important tool for a group of central banks to minimize the group's social loss. Financial aid channels social loss of a central bank with lower realignment cost to another central bank with higher realignment cost, which enables countries within a group to defend the cooperative equilibrium from a larger size of speculative attack. It explains the formation of numerous regional and international organizations aim at promoting financial stability, either regionally or globally. The most recent development is that China, South Korea, Japan and other members of Association of Southeast Asian Nations (ASEAN) have agreed to create a pool of US$80 billion to help protecting stability of members' currencies[13].

Unfortunately, this paper reveals financial aid may fails to defend the cooperative equilibrium in some cases. Obviously, relatively large size of speculative attack comparing with the realignment cost of the financial aid provider can be one case. In this case, the financial aid provider choose to fall into

[13] Japan, S. Korea, China Mull $80 Billion Reserve Pool, Bloomberg News, 4 May 2008 (http://www.bloomberg.com/apps/news?pid=20601080&sid=awCGhWbRKOXY)

competitive devaluations instead of providing sufficient financial aid to help the other central bank to defend its currency. May be more surprisingly, high realignment cost of the financial aid provider does not guarantee a successful cooperation as well. Such high realignment cost makes the fixed exchange rate policy a dominant strategy to the financial aid provider, sheltering its social loss from the impact of realignment cost and thus preventing it to provide more financial aid.

Special features of the game, the multiple critical values for the domestic bank's policy response and the effect of the "weakest link" in defending a country's fixed exchange rate may shed light on future study. The model and games in this paper is highly stylized and much further work can be done to extend the model, for example, central banks with asymmetric tradeoff. Surely this paper is not the whole and prefect story for exchange rate policy coordination in financial forms but hopefully, it can arouse interest in future study.

7. References

Angeletos, George-Marios, Christian Hellwig and Alessandro Pavian (2006), "Signaling in a Global Game: Coordination and Policy Traps", *The Journal of Political Economy*, 114(3), 452-484.

Chan, Alex and Nai-Fu Chen (2003), "A Theory of Currency Board with Irrevocable Commitments", *International Review of Finance*, 4(3-4), 125-170.

Fernald, John, Hali Edison and Prakash Loungani (1999), "Was China the First Domino? Assessing Links Between China and the Rest of Emerging Asia", *Journal of International Money and Finance*, 18, 515-535

Flood, Robert and Peter Garber (1984), "Collapsing Exchange Rate Regimes: Some Linear Examples", *Journal of International Economics,* 17, 1-13.

Hong, Kiseok and Aaron Tornell (2003), "Recovery from a currency crisis: some stylized facts", *Journal of Development Economics,* 76, 71-96

Kasa, Kenneth and Chan Huh (2001), "A Dynamic Model of Export Competition, Policy Coordination, and Simultaneous Currency Collapse", *Review of International Economics,* 9(1), 68-80.

Krugman, Paul (1979), "A model of Balance of Payments Crises", *Journal of Money, Credit, and Banking*, 11, 311-325.

Morris, Stephen and Hyun-Song Shin (1998), "Unique Equilibrium in a Model of Self-Fulfilling Currency Attacks", *American Economic Review*, 88(3), 587-597

Obstfeld Maurice (1986), "Rational and Self-Fulfilling Balance-of-Payments Crises", *American Economic Review,* 76(1), 72-81

Obstfeld, Maurice (1994), "The Logic of Currency Crises", *NBER Working Paper,* No. 4640, February.

Obstfeld, Maurice (1996), "Models of currecy crises with self-fulfilling features", *European Economic Review,* 40(3-5), 1037-1047.

Rotemberg, Julio J. and Garth Saloner (1986), "A Supergame – Theoretic Model of Price Wars during Booms", *American Economic Review*, 76, 390-407

8. Appendix

8.1. Change in Price Level and Exchange Rate

Given that domestic price level of country i (P_i) is composed of both prices of domestic goods (P_D) and imported goods (P_F) in the following form:

$$P_i = P_D^{1-\beta} P_F^{\beta},$$

where β is the share of imported goods in the domestic consumption of country i and

$$P_F = E_i \hat{P}_F,$$

where E_i is the is the direct quote of the country i's currency and $\hat{P}_F$ is the price of imported goods in foreign currency. From this,

$$P_i = P_D^{1-\beta} (E_i \hat{P}_F)^{\beta}$$

$$\log P_i = (1-\beta)\log P_D + \beta \log E_i + \beta \log \hat{P}_F$$

$$\frac{d\log P_i}{dt} = (1-\beta)\frac{d\log P_D}{dt} + \beta\frac{d\log E_i}{dt} + \beta\frac{d\log \hat{P}_F}{dt}$$

$$\frac{d\log P_i}{dP_i}\frac{dP_i}{dt} = (1-\beta)\frac{d\log P_D}{dP_D}\frac{dP_D}{dt} + \beta\frac{d\log E_i}{dE_i}\frac{dE_i}{dt} + \beta\frac{d\log \hat{P}_F}{d\hat{P}_F}\frac{d\hat{P}_F}{dt}$$

Given that $\frac{dP_D}{dt} = 0, \frac{d\hat{P}_F}{dt} = 0$

$$\frac{\dot{P}_i}{P_i} = \beta\frac{\dot{E}_i}{E_i}$$

8.2. Optimization of Depreciation Rate

Putting (3) and (4) into (1),

$$L_i = \frac{1}{2}[\alpha(e_i - e_j) - \bar{y}]^2 + \frac{\gamma}{2}e_i^2$$

To minimize loss with depreciation rate of its currency, country *i* should

$$\frac{dL_i}{de_i} = 0$$

$$[\alpha(e_i - e_j) - \bar{y}]\alpha + \gamma e_i = 0$$

$$(\alpha^2 + \gamma)e_i = \alpha^2 e_j + \alpha\bar{y}$$

$$e_i = \frac{\alpha\bar{y} + \alpha^2 e_j}{\alpha^2 + \gamma}$$

8.3. Social Loss for Unilateral Devaluation

Putting (3) and (4) into (1) with e_i *as (7) and* e_j=0,

$$L_i = \frac{1}{2}\left[\alpha\left(\frac{\alpha}{\alpha^2+\gamma}\bar{y}\right)-\bar{y}\right]^2+\frac{\gamma}{2}\left(\frac{\alpha}{\alpha^2+\gamma}\bar{y}\right)^2$$

$$L_i = \frac{1}{2}\left[\frac{\gamma}{\alpha^2+\gamma}\bar{y}\right]^2+\frac{1}{2}\bar{y}^2\left[\frac{\alpha^2\gamma}{\left(\alpha^2+\gamma\right)^2}\right]$$

$$L_i = \frac{1}{2}\bar{y}^2\left[\frac{\gamma^2}{\left(\alpha^2+\gamma\right)^2}+\frac{\alpha^2\gamma}{\left(\alpha^2+\gamma\right)^2}\right]$$

$$L_i = \frac{1}{2}\bar{y}^2\left(\frac{\gamma}{\alpha^2+\gamma}\right)$$

8.4. Social Loss under Foreign Unilateral Devaluation

Putting (3) and (4) into (1) with e_i=0 and e_j as (7),

$$L_i = \frac{1}{2}\left[\alpha\left(-\frac{\alpha}{\alpha^2+\gamma}\bar{y}\right)-\bar{y}\right]^2$$

$$L_i = \frac{1}{2}\left[\frac{\alpha^2}{\alpha^2+\gamma}\bar{y}+\bar{y}\right]^2$$

$$L_i = \frac{1}{2}\bar{y}^2\left(1+\frac{\alpha^2}{\alpha^2+\gamma}\right)$$

8.5. Social Loss for Competitive Devaluations

Putting e_i=e_j in (7),

$$e_i = \frac{\alpha\bar{y}}{\alpha^2+\gamma} + \frac{\alpha^2 e_i}{\alpha^2+\gamma}$$

$$\frac{\gamma e_i}{\alpha^2+\gamma} = \frac{\alpha\bar{y}}{\alpha^2+\gamma}$$

$$e_i = \frac{\alpha}{\gamma}\bar{y}$$

Putting (3) and (4) into (1) with $e_i = e_j = \frac{\alpha}{\gamma}\bar{y}$,

$$L_i = \frac{1}{2}\bar{y}^2 + \frac{\gamma}{2}\left(\frac{\alpha}{\gamma}\bar{y}\right)^2$$

$$L_i = \frac{1}{2}\bar{y}^2 + \frac{1}{2}\bar{y}^2\frac{\alpha^2}{\gamma}$$

$$L_i = \frac{1}{2}\bar{y}^2$$

$$L_i = \frac{1}{2}\bar{y}^2\left(1+\frac{\alpha}{\gamma}\right)$$

8.6. Impact of φ on λ^l

Differentiation of (20):

$$\frac{d}{d\varphi}\left[\frac{1}{2}\bar{y}^2\left(\frac{1+2\varphi}{1+\varphi}\right)^2-(1+\varphi)\right]$$

$$=\frac{1}{2}\bar{y}^2\left\{2\left(\frac{1+2\varphi}{1+\varphi}\right)\left[\frac{2}{1+\varphi}-\frac{1+2\varphi}{(1+\varphi)^2}\right]-1\right\}$$

$$=\frac{1}{2}\bar{y}^2\left[\frac{2+4\varphi}{(1+\varphi)^3}-1\right]$$

$$\frac{d^2}{d\varphi^2}\left[\frac{1}{2}\bar{y}^2\left(\frac{1+2\varphi}{1+\varphi}\right)^2-(1+\varphi)\right]$$

$$=\frac{1}{2}\bar{y}^2\left[\frac{4}{(1+\varphi)^3}-\frac{3(2+4\varphi)}{(1+\varphi)^4}\right]$$

$$=-\bar{y}^2\left[\frac{1+4\varphi}{(1+\varphi)^4}\right]\leq 0 \qquad \forall\varphi\geq 0$$

Value of φ that maximize (20):

$$\frac{d}{d\varphi}\left[\frac{1}{2}\bar{y}^2\left(\frac{1+2\varphi}{1+\varphi}\right)^2-(1+\varphi)\right]=0$$

$$\frac{1}{2}\bar{y}^2\left[\frac{2+4\varphi}{(1+\varphi)^3}-1\right]=0$$

$$2+4\varphi=(1+\varphi)^3$$

$$\varphi^3+3\varphi^2-\varphi-1=0$$

$$\therefore\varphi=0.675$$

8.7. Optimization Benefit under different foreign policy

$$\lambda^f - \lambda^l$$

$$= \frac{1}{2}\bar{y}^2\left(\frac{\alpha^2}{\alpha^2+\gamma}\right) - \frac{1}{2}\bar{y}^2\left[\left(1+\frac{\alpha^2}{\alpha^2+\gamma}\right)^2 - \left(1+\frac{\alpha^2}{\gamma}\right)\right]$$

$$= \frac{1}{2}\bar{y}^2\left[\left(\frac{\alpha^2}{\alpha^2+\gamma}\right) - \left(1+\frac{\alpha^2}{\alpha^2+\gamma}\right)^2 + \left(1+\frac{\alpha^2}{\gamma}\right)\right]$$

$$= \frac{1}{2}\bar{y}^2\left[\frac{\alpha^2}{\gamma} - \left(\frac{\alpha^2}{\alpha^2+\gamma}\right) - \left(\frac{\alpha^2}{\alpha^2+\gamma}\right)^2\right]$$

$$= \frac{1}{2\gamma(\alpha^2+\gamma)^2}\bar{y}^2\left[\alpha^2(\alpha^2+\gamma)^2 - \alpha^2\gamma(\alpha^2+\gamma) - \alpha^4\gamma\right]$$

$$= \frac{1}{2\gamma(\alpha^2+\gamma)^2}\bar{y}^2\left[\alpha^2(\alpha^2+\gamma)^2 - \alpha^2\gamma(\alpha^2+\gamma) - \alpha^4\gamma\right]$$

$$= \frac{\alpha^6}{2\gamma(\alpha^2+\gamma)^2}\bar{y}^2 \geq 0 \qquad \forall \alpha \text{ and } \gamma \geq 0$$

8.8. The Complete Two-Country Game with Financial Aid

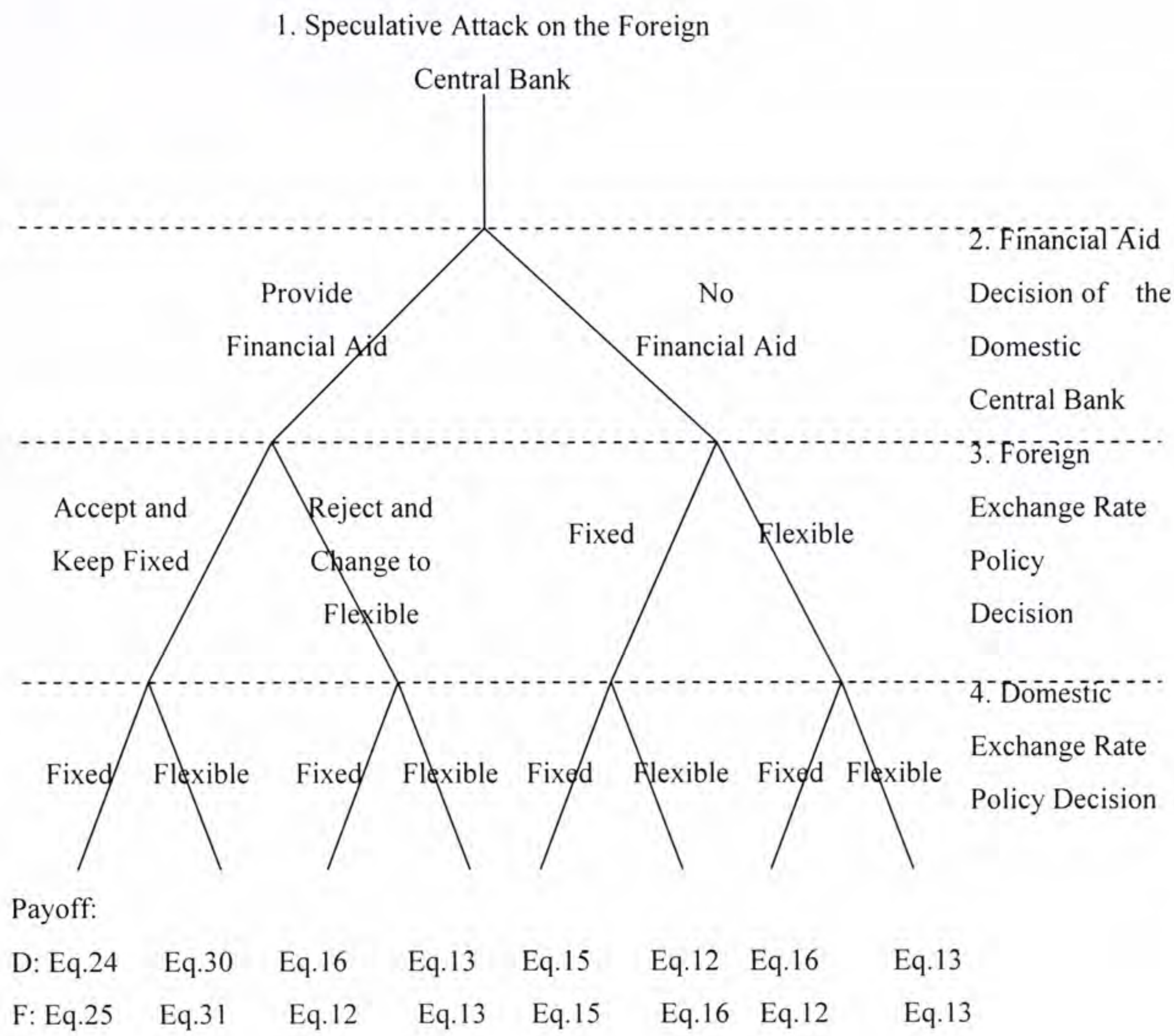

Figure 6 – The complete two-country game with financial aid.

Social loss function for unilateral devaluation with financial aid is:

$$\frac{1}{2}\bar{y}^2\left(\frac{\gamma}{\alpha^2+\gamma}\right)+C_i+F \text{ or } L_i^{lf}+F\,. \tag{30}$$

Social loss for maintaining fixed exchange rate under foreign unilateral devaluation with financial aid is:

$$\frac{1}{2}\bar{y}^2\left(1+\frac{\alpha^2}{\alpha^2+\gamma}\right)^2+S_i^l-F \text{ or } L_i^{fl}-F\,. \tag{31}$$